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## ORBITS AND PULSATIONS OF THE CLASSICAL $\zeta$ AURIGAE BINARIES

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### ABSTRACT

We have derived new orbits for  $\zeta$  Aur, 32 Cyg, and 31 Cyg with observations from the Tennessee State University (TSU) Automatic Spectroscopic Telescope, and used them to identify nonorbital velocities of the cool supergiant components of these systems. We measure periods in those deviations, identify unexpected long-period changes in the radial velocities, and place upper limits on the rotation of these stars. These radial-velocity variations are not obviously consistent with radial pulsation theory, given what we know about the masses and sizes of the components. Our concurrent photometry detected the *nonradial* pulsations driven by tides (ellipsoidal variation) in both  $\zeta$  Aur and 32 Cyg, at a level and phasing roughly consistent with simple theory to first order, although they seem to require moderately large gravity darkening. However, the K component of 32 Cyg must be considerably bigger than expected, or have larger gravity darkening than  $\zeta$  Aur, to fit its amplitude. However, again there is precious little evidence for the normal radial pulsation of cool stars in our photometry.  $H\alpha$  shows some evidence for chromospheric heating by the B component in both  $\zeta$  Aur and 32 Cyg, and the three stars show among them a meager  $\sim 2$ – $3$  outbursts in their winds of the sort seen occasionally in cool supergiants. We point out two fundamental questions in the interpretation of these stars: (1) whether it is appropriate to model the surface brightness as gravity darkening and (2) whether much of the nonorbital velocity structure may actually represent changes in the convective flows in the stars' atmospheres.

*Subject headings:* binaries: spectroscopic — stars: late-type — stars: oscillations

*Online material:* machine-readable tables

### 1. INTRODUCTION

Our detailed knowledge of stars in the main sequence comes from analyses of eclipsing double-lined spectroscopic binaries. Solutions to light and velocity curves of such objects can define masses and radii of the component stars well enough to challenge the details of calculated internal structure and evolution. In contrast, defining the basic properties of evolved stars is normally much more difficult. The long periods and correspondingly large separations of binaries containing them make eclipses unlikely, and the existing binaries tend to be only single-lined. The  $\zeta$  Aur binaries, however, with their eclipses and composite spectra, give us a unique opportunity to determine physical properties of a few massive supergiant stars reliably in the same way we can for many main-sequence stars. A good example of this is the way Bennett et al. (1996) defined the properties of  $\zeta$  Aur. Wright (1970) discussed these systems, particularly the three classical systems  $\zeta$  Aur, 31 Cyg, and 32 Cyg, all three of which have supergiant K primaries paired with B stars close to the main sequence. Table 1 gives their fundamental properties.

Most close binaries have circular orbits, although all possible eccentricities seem equally likely among newborn systems, at least those with longer periods (e.g., Abt 2006). The three classical  $\zeta$  Aur systems all have sizable eccentricities. For this reason, they ought to be subject to certain binary proximity effects in ways other stars are not. For instance, they will be subject to a nonradial pulsation driven by the variable tidal distortion inevitable in an eccentric binary (Cowling 1941; Eaton 2008; Sepinsky et al. 2007). This phenomenon is equivalent to the ellipsoidal variation from the equilibrium tidal distortion in circular, synchronously

rotating binaries. Because of the different orientations of their orbits, the two closer systems  $\zeta$  Aur and 32 Cyg would manifest different phase dependence of this effect in ways giving clues about the internal properties of supergiants. Guinan & McCook (1979) claimed to have detected this phenomenon in 32 Cyg. Wilson (1979) included a theory for it in the Wilson-Devinney code for calculating binary light curves. The other major proximity effect, the so-called reflection effect, might be detectable in these systems as well.

All the cool giants and supergiants seem to be variable, probably through radial pulsations. Henry et al. (2000) argued that all stars to the red of the Linsky-Haisch coronal dividing line are pulsational variables. Even the red giants to the blue of it are variable given precise radial velocities (Walker et al. 1989). The components of  $\zeta$  Aur binaries would be expected to manifest the pulsations of similar supergiants. Such pulsations would be in addition to the aforementioned proximity effects. Differences in their pulsational periods *might* give us an idea of how mass is distributed in their interiors.

$\zeta$  Aur itself is the most interesting of the three classical systems in terms of its binary interactions. It has the shortest period (970 days) and biggest eccentricity ( $e \sim 0.4$ ), and these qualities make it most interesting for looking for the effects of a tidally driven nonradial pulsation. Griffin (2005) discussed the orbit recently, using all the many radial velocities then available. Why, then, should we waste our time redoing his analysis? Our data are several times as precise, cover one orbit continuously, and thereby begin to show coherent deviations of the K star from its orbital velocity.

We will (1) improve the orbital elements for two of the three classical  $\zeta$  Aur binaries, (2) assess the rotation of these stars, (3) model the ellipsoidal light variations in order to interpret the driven pulsations of the cooler components of these systems, (4) look for the intrinsic (radial) pulsations of these stars and

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TABLE 1  
THE CLASSICAL  $\zeta$  AUR SYSTEMS

| Star<br>(1)      | Spectrum<br>(2) | Period<br>(days)<br>(3) | $a$<br>( $R_{\odot}$ )<br>(4) | $i$<br>(deg)<br>(5) | $R_1$<br>( $R_{\odot}$ )<br>(6) | $r$<br>( $R_1/a$ )<br>(7) | $M_1$<br>( $M_{\odot}$ )<br>(8) | $M_2$<br>( $M_{\odot}$ )<br>(9) | References<br>(10)    |
|------------------|-----------------|-------------------------|-------------------------------|---------------------|---------------------------------|---------------------------|---------------------------------|---------------------------------|-----------------------|
| $\zeta$ Aur..... | K4 Ib + B5 V    | 972                     | 905                           | 87.0                | 148                             | 0.163                     | 5.8                             | 4.8                             | Bennett et al. (1996) |
| HD 32068.....    |                 |                         |                               |                     |                                 |                           |                                 |                                 |                       |
| 31 Cyg.....      | K4 Ib + B3–4    | 3784                    | 2710                          | 87.2                | 197                             | 0.073                     | 11.7                            | 7.1                             | Eaton (1993b)         |
| HD 192577.....   |                 |                         |                               |                     |                                 |                           |                                 |                                 | Eaton & Bell (1994)   |
| Cyg 32.....      | K4–5 Ib + B6–7  | 1148                    | 1130                          | 78.6                | 175                             | 0.155                     | 9.7                             | 4.8                             | Eaton (1993a)         |
| HD 192909.....   |                 |                         |                               |                     |                                 |                           |                                 |                                 |                       |

NOTES.—Spectral types are from Wright (1970). Other quantities are from the cited references in Col. (10).

use them to restrict the radii, and (5) look for evidence of proximity effects and other variation in  $H\alpha$ .

## 2. OBSERVATIONS

Observations consist of new spectra and photometry for the three classical systems, spanning roughly 3.5 yr since 2004. All these data come from the completely automatic observatory Tennessee State University (TSU) maintains at Fairborn Observatory, a private site in southern Arizona (Eaton et al. 1996).

### 2.1. Spectra

We observed  $\zeta$  Aur, 31 Cyg, and 32 Cyg between JD 2,452,860 and 2,454,200, obtaining echelle spectra of roughly 30,000 resolution, with the TSU 2 m Automatic Spectroscopic Telescope (AST; Eaton & Williamson 2004, 2007). This set consists of 302, 217, and 348 useful spectra, respectively, for the three stars. We reduced and analyzed them with standard pipeline techniques to derive radial velocities and equivalent widths of  $H\alpha$ . These measurements are available electronically as Table 2. Listed are: (1) HJD, the Heliocentric Julian Date of observation (minus 2,400,000); (2)  $RV_{\text{cool}}$ , the radial velocity of the K star; (3) EW1, an equivalent width of  $H\alpha$  absorption; and (4) EW2, an equivalent width of enhanced absorption in the blue wing of  $H\alpha$ . Column (5) is a tag identifying the star by its HD number. Missing data in this table are identified with a “9.999.”

The measured velocities from the AST have a formal external error of 0.10–0.11  $\text{km s}^{-1}$  and are  $0.35 \pm 0.09 \text{ km s}^{-1}$  more negative than the IAU radial-velocity system (Eaton & Williamson 2007). The velocities in Table 2 are the raw velocities without the  $+0.35 \text{ km s}^{-1}$  correction to the IAU system. Values of systemic

TABLE 2  
SAMPLE SPECTROSCOPIC DATA

| HJD<br>(2,400,000+)<br>(1) | $RV_{\text{cool}}$<br>( $\text{km s}^{-1}$ )<br>(2) | EW1<br>( $\text{\AA}$ )<br>(3) | EW2<br>( $\text{\AA}$ )<br>(4) | Star<br>(5) |
|----------------------------|---|--------------------------------|--------------------------------|-------------|
| 52,861.9477.....           | 0.11  | 1.334                          | 0.113                          | HD 32068    |
| 52,863.8894.....           | 0.02  | 1.496                          | 0.152                          | HD 32068    |
| 52,895.9664.....           | 3.60  | 1.426                          | 0.167                          | HD 32068    |
| 52,896.9640.....           | 3.70  | 1.407                          | 0.149                          | HD 32068    |
| 52,897.9219.....           | 3.87  | 1.446                          | 0.161                          | HD 32068    |
| 52,898.9425.....           | 3.94  | 1.447                          | 0.162                          | HD 32068    |
| 52,899.9319.....           | 3.93  | 1.419                          | 0.154                          | HD 32068    |

NOTES.—Table 2 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.

velocities,  $\gamma$ , from our orbital solutions, listed in Table 4, are transformed to the IAU system.

Our  $H\alpha$  data consist of observed equivalent widths in a wide band (6561.3–6565.05  $\text{\AA}$  in the rest frame of the star), EW1, designed to measure the total absorption in the normal profile of such a star, and a narrow band (6559.7–6561.3  $\text{\AA}$ ), EW2, to detect enhancements of the blue wing of the profile that may signal episodic mass ejections in a star’s wind. In measuring the spectra, we adjusted the continuum to a common level by defining 13 pseudocontinuum points in the range 6522–6600  $\text{\AA}$ , automatically measured their levels in the spectra, and renormalizing the spectra to line segments between those points. The EWs depend on the two points at 6559.3 and 6568.1  $\text{\AA}$ . Several hundred spectra of the K giants  $\alpha$  Tau,  $\alpha$  Boo, and  $\alpha$  Ari, which ought to be  $\sim$ constant in  $H\alpha$ , give standard deviations per measurement of 0.046 and 0.020  $\text{\AA}$ , respectively, for EW1 and EW2. We shall use these values as the uncertainties of measurement.

### 2.2. Photometry

We also collected  $BV$  observations of the three stars with the TSU 0.4 m Automatic Photometric Telescope (APT), obtaining measurements over complete cycles of both  $\zeta$  Aur and 32 Cyg and 4.0 yr for 31 Cyg. These measurements consist of nightly means of differential measurements with respect to a comparison star, HD 34412 for  $\zeta$  Aur and HD 192985 for both 31 and 32 Cyg. The check star for  $\zeta$  Aur was HD 30834, with 32 Cyg observed as a check star for 31 Cyg. These data should have an external error near 0.004 mag (Henry 1995). They are available electronically as Table 3. Data listed are HJD  $-2,400,000$ ,  $(\Delta U, \Delta B, \Delta V)_{\text{variable}}$ ,  $(\Delta U, \Delta B, \Delta V)_{\text{check}}$ —when available, and HD number of the star. We identify missing data, such as the nonexistent  $\Delta U$ ’s, with a “99.999”. This arrangement preserves the format of photometric data available on our internet site.

## 3. ANALYSIS

We shall analyze the three stars to find out how their radial velocities deviate from purely orbital motion and combine these results with photometry to assess what forms the pulsations and proximity effects take in them.

### 3.1. Deviations from Orbital Velocities

The great precision of our AST data lets us solve velocity curves of these three long-period binaries and look for deviations from elliptical orbits. Table 4 gives the results for the three stars, listing the usual spectroscopic elements. For 31 Cyg we have only about one-third of a full orbital cycle of data, so we have combined our data with those listed by Wright & Huffman (1968) and weighted all the data equally. The solution to this combined data set is the same as Wright & Huffman’s to within the putative

TABLE 3  
SAMPLE PHOTOMETRIC DATA

| HJD<br>(2,400,000+)<br>(1) | $\Delta U_{\text{var}}$<br>(2) | $\Delta B_{\text{var}}$<br>(3) | $\Delta V_{\text{var}}$<br>(4) | $\Delta U_{\text{chk}}$<br>(5) | $\Delta B_{\text{chk}}$<br>(6) | $\Delta V_{\text{chk}}$<br>(7) | Star<br>(8) |
|----------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|-------------|
| 52,895.9881.....           | 99.999                         | -0.327                         | -0.942                         | 99.999                         | 0.886                          | 0.085                          | HD 32068    |
| 52,926.0065.....           | 99.999                         | -0.324                         | -0.934                         | 99.999                         | 0.874                          | 0.079                          | HD 32068    |
| 52,930.0235.....           | 99.999                         | -0.322                         | -0.933                         | 99.999                         | 0.876                          | 0.081                          | HD 32068    |
| 52,931.0199.....           | 99.999                         | -0.324                         | -0.939                         | 99.999                         | 0.877                          | 0.077                          | HD 32068    |
| 52,932.0135.....           | 99.999                         | -0.327                         | -0.939                         | 99.999                         | 0.878                          | 0.079                          | HD 32068    |
| 52,933.0132.....           | 99.999                         | -0.327                         | -0.940                         | 99.999                         | 0.876                          | 0.080                          | HD 32068    |
| 52,934.0214.....           | 99.999                         | -0.326                         | -0.938                         | 99.999                         | 0.871                          | 0.069                          | HD 32068    |

NOTES.—Table 3 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.

errors. The difference between the spectroscopic period we have derived and the photometric period (1.8 days) corresponds to a shift of  $+0.21 \text{ km s}^{-1}$  of Wright & Huffman’s velocities with respect to ours. This shift gives a flavor of the kind of uncertainties that indeterminate zero-point shifts introduce into orbital analyses. The values of the major elements ( $K$ ,  $e$ , and  $\omega$ ) for both  $\zeta$  Aur and 32 Cyg agree with previous determinations to within the likely errors of those determinations. In particular, they agree with Griffin’s (2005) values for  $\zeta$  Aur to within  $2 \sigma$  of his formal errors. For 32 Cyg, they should be a significant improvement on the elements of Wright (1970), with which they agree to within the likely errors of that analysis. This excellent agreement means that the shapes and orientations of the orbits of both stars are known well enough to support rigorous analyses of their atmospheric eclipses, driven nonradial pulsations, and eclipse timings.

Figure 1 shows the velocity curves of the three stars. All three are obviously variable, with nonorbital shifts superimposed on the dominant orbital velocities. Figure 2 shows the time dependence of these deviations, which can be rather extreme. The 250 day,  $0.75 \text{ km s}^{-1}$ , deviation of 31 Cyg around 53,700, for instance, if pulsational, would correspond to a  $23 R_{\odot}$  change in the stellar radius, about 12%, even without any allowance for foreshortening. Alternatively, it could represent some sort of truly global circulation. The variation seen in Figure 2 seems cyclic on time-scales of 100–300 days, so one might suspect that some of it could be seasonal observational effects. However, the major effects do not correlate very well in 31 and 32 Cyg, which we observed over the same observing season, occasionally on the same nights, and there is absolutely no hint of such effects in velocities of HD 14214 at levels above  $0.05 \text{ km s}^{-1}$  (Eaton & Williamson 2007). Three other K supergiants observed over roughly the same time interval,  $\epsilon$  Peg (K2 Ib),  $\xi$  Cyg (K4–5 Ib–II), and 63 Cyg (K4 Ib–IIa), show long-term variations at least as great as the

three binaries, although we have far fewer data for these single stars.

We have used two techniques to look for periodicity in the residuals. In the first, using a program written by D. S. Hall, we fit sine curves for a spectrum of periods to the data [ $\Delta RV = A \sin(2\pi \text{HJD}/P + \phi)$ ] and identified minima of  $\chi^2$  of these fits as possible periods. In the second, we applied the techniques of Vaniček (1971), as we have for the multiperiodic  $\gamma$  Doradus stars (e.g., Kaye et al. 1999; Henry et al. 2001). This second approach lets us reliably find multiple periodicities without “prewhitening,” an advantage, especially in the low-frequency domain. We searched for periods in the range 1–1200 days. Both methods identified essentially the same periods, but, because the second method gives more systematic results, we will use them in the following analysis. Table 5 lists the periods found and amplitudes of sinusoids fit to the data for them. If the velocity variation represents a radial pulsation, we may integrate the (sinusoidal) variation over half a cycle to get the total excursion in radius,  $\Delta R = \xi AP/\pi$ , where  $\xi \approx 1.35$  is a correction for the fact that expansion of most of the disk is only partially in the line of sight (e.g., Gray & Stevenson 2007). The periods derived here likely reflect the timescales of some physical phenomena in these stars but not truly coherent long-duration pulsations. This is especially so for the longest periods, those comparable to the  $\sim 1200$  day duration of our observations. Additional tests for shorter periods (0.03–1.0 days) with the method of Vaniček found none, as expected.

### 3.2. Rotation

If the cool components of these binaries were rotating synchronously, they would have significant rotational velocities,  $v \sin i = (K_1 + K_2)R_1/a$  for synchronous rotation with the usual assumptions about orientation of the motions. For pseudosynchronous rotation (Hut 1981; Hall 1986), the velocity would be

TABLE 4  
SPECTROSCOPIC ORBITS

| HD<br>(1)        | Period<br>(days)<br>(2) | $T^a$<br>(HJD–2,400,000)<br>(3) | K<br>( $\text{km s}^{-1}$ )<br>(4) | $\gamma^b$<br>( $\text{km s}^{-1}$ )<br>(5) | $e$<br>(6)          | $\omega$<br>(deg)<br>(7) | References<br>(8) |
|------------------|-------------------------|---------------------------------|------------------------------------|---|---------------------|--------------------------|-------------------|
| $\zeta$ Aur..... | (972.162)               | $53,039.9 \pm 0.10$             | $23.17 \pm 0.02$                   | $10.81 \pm 0.01$                            | $0.3973 \pm 0.0007$ | $328.9 \pm 0.13$         | This paper        |
| 31 Cyg.....      | $3786.1 \pm 0.7$        | $52,372.8 \pm 2.2$              | $13.78 \pm 0.13$                   | $-7.41 \pm 0.08$                            | $0.224 \pm 0.006$   | $206.4 \pm 1.4$          | This paper        |
| 32 Cyg.....      | (1147.80)               | $53,796.9 \pm 0.28$             | $16.64 \pm 0.03$                   | $-7.45 \pm 0.02$                            | $0.3126 \pm 0.0014$ | $222.1 \pm 0.3$          | This paper        |

NOTES.—Values in parentheses are assumed values taken from the literature. Periods are generally from Batten et al. (1989).

<sup>a</sup> Periastron passage.

<sup>b</sup> Velocity on IAU system.

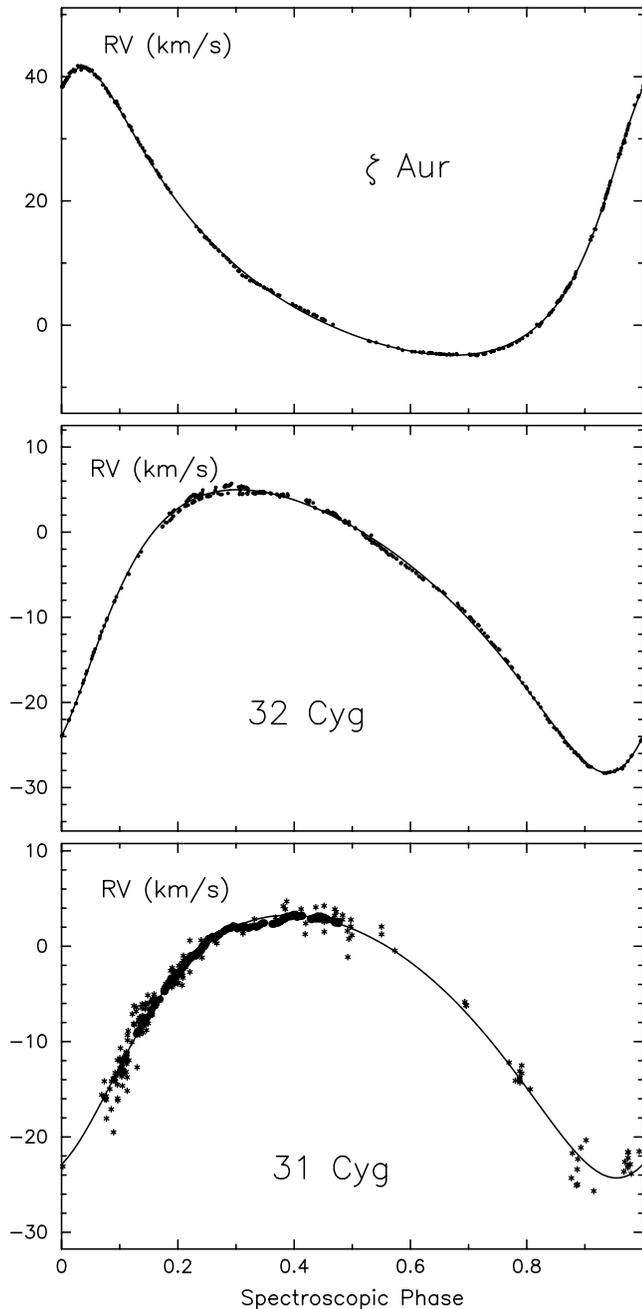


FIG. 1.—Velocity curves of  $\zeta$  Aur binaries. Velocities from the AST are shown as dots. The curves represent the orbital elements in Table 4. The asterisks for 31 Cyg are the velocities listed by Wright & Huffman (1968).

even bigger. For  $\zeta$  Aur, the values would be  $8.3$  and  $16.6 \text{ km s}^{-1}$ , respectively, with the enhancement for pseudosynchronous rotation calculated with Hut's equation (42). Rotational velocities, hence line broadening, for the other two systems would be less, an unobservable  $v \sin i = 2.71$  and  $3.6 \text{ km s}^{-1}$  for 31 Cyg, for example, although it should be observable for 32 Cyg at  $v \sin i = 6.50$  and  $10.4$  (or  $10$  and  $15$  if we adopt the much bigger relative radius implied by tidal distortion). If  $\zeta$  Aur were rotating pseudosynchronously, we could easily detect it by comparing profiles of its metallic lines with those in 31 Cyg and other, single, stars. A couple of well-exposed single spectra of  $\zeta$  Aur and 31 Cyg do not show shallower, hence broader, lines in  $\zeta$  Aur than in 31 Cyg, nor do the spectra of  $\zeta$  Aur binaries and other K supergiants plotted by Eaton (1995; his Fig. A19) show an apparent

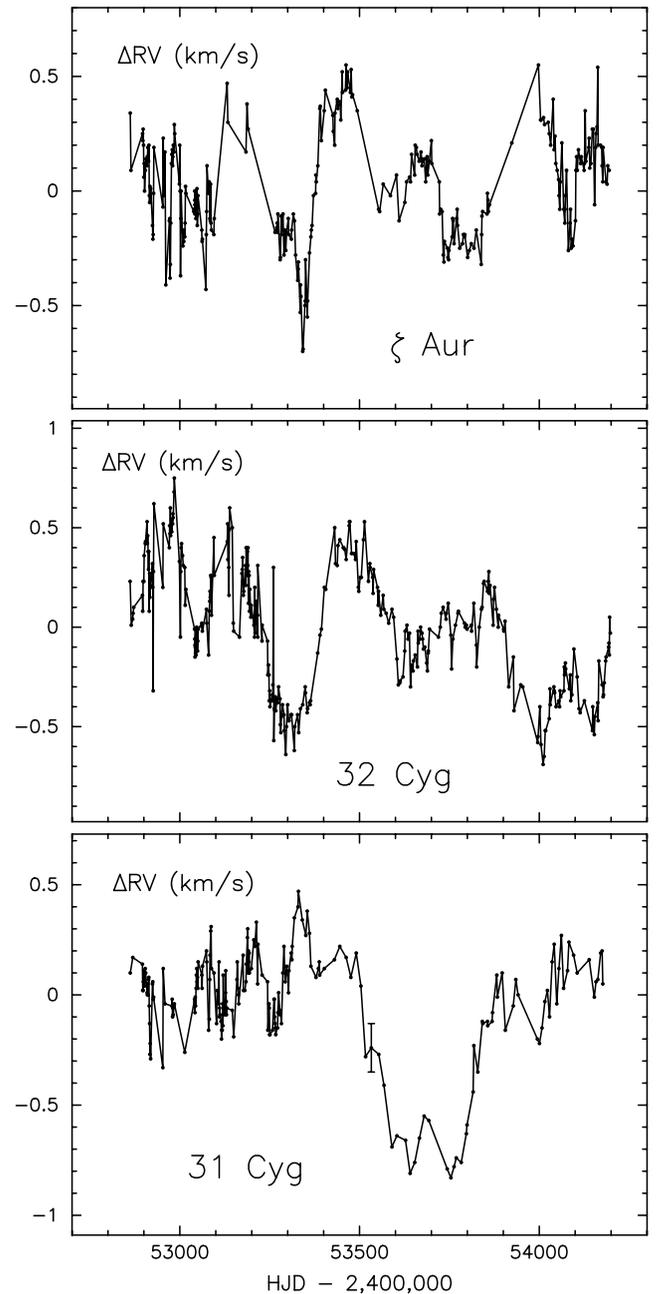


FIG. 2.—Deviations from the fitted elliptical orbits. Data from the AST are shown as dots. We have connected the successive data points with line segments because we think this makes it easier to see and assess the shorter term variability. The error bar plotted in the middle of the data string for 31 Cyg shows the level of uncertainty expected for all these data.

difference in depths of strong metallic lines. To get an idea of the magnitude of the expected broadening, we artificially broadened a spectrum of the K2 Ib supergiant  $\epsilon$  Peg to  $v \sin i = 8.3$  and  $16.6 \text{ km s}^{-1}$  and compared the line profiles in the broadened spectra with unbroadened profiles. Both values gave measurably shallower lines, by 10% and 30%, respectively.

To quantify this result, we have looked at the strengths and depths of strong metallic lines in three other cool supergiants,  $\epsilon$  Peg,  $\xi$  Cyg, and 63 Cyg, plotting them up with composite spectra for the three  $\zeta$  Aur binaries for phases when the K supergiants were not illuminated by their B companions. Depths of strong Fe I lines in the three single stars vary by  $\sim 2\%$ . The lines in  $\zeta$  Aur itself in these composites were actually somewhat

TABLE 5  
POSSIBLE PERIODS IN VELOCITY RESIDUALS

| Star<br>(1)       | Period<br>(days)<br>(2) | $A$<br>( $\text{km s}^{-1}$ )<br>(3) | $\Delta R$<br>( $R_{\odot}$ )<br>(4) | $Q$<br>(days)<br>(5) |
|-------------------|-------------------------|--------------------------------------|--------------------------------------|----------------------|
| $\zeta$ Aur ..... | $252.3 \pm 2.4$         | $0.111 \pm 0.011$                    | 1.5                                  | 0.34                 |
|                   | $431.6 \pm 5.0$         | $0.131 \pm 0.011$                    | 3.0                                  | 0.58                 |
|                   | $658.8 \pm 10.2$        | $0.078 \pm 0.010$                    | 2.7                                  | 0.88                 |
|                   | $186.8 \pm 0.9$         | $0.063 \pm 0.009$                    | 0.6                                  | 0.25                 |
| 31 Cyg .....      | $1062 \pm 26$           | $0.287 \pm 0.016$                    | 16                                   | 1.31                 |
|                   | $542 \pm 10$            | $0.108 \pm 0.012$                    | 3.1                                  | 0.67                 |
|                   | $125.2 \pm 0.5$         | $0.081 \pm 0.009$                    | 0.5                                  | 0.15                 |
|                   | $347.2 \pm 3.0$         | $0.050 \pm 0.009$                    | 0.9                                  | 0.43                 |
|                   | $166.4 \pm 0.8$         | $0.063 \pm 0.009$                    | 0.6                                  | 0.20                 |
| 32 Cyg .....      | $201.1 \pm 1.0$         | $0.045 \pm 0.009$                    | 0.5                                  | 0.25                 |
|                   | $721 \pm 11$            | $0.160 \pm 0.011$                    | 6.1                                  | 0.97                 |
|                   | $314.8 \pm 3.5$         | $0.173 \pm 0.011$                    | 2.9                                  | 0.42                 |
|                   | $443.5 \pm 4.2$         | $0.144 \pm 0.010$                    | 3.4                                  | 0.60                 |
|                   | $1483 \pm ??$           | $0.128 \pm 0.010$                    | 10                                   | 2.00                 |
|                   | $234.4 \pm 1.7$         | $0.100 \pm 0.010$                    | 1.2                                  | 0.32                 |
|                   | $153.4 \pm 0.4$         | $0.065 \pm 0.010$                    | 0.5                                  | 0.21                 |
|                   | $109.6 \pm 0.2$         | $0.064 \pm 0.009$                    | 0.4                                  | 0.15                 |

shallower than in most of the other stars, but by only of order 3%. This implies a rotational velocity  $\sim 30\%$  synchronous, or  $2.5 \text{ km s}^{-1}$ . On the other hand, 32 Cyg, which ought to be rotating about as fast as  $\zeta$  Aur, has lines as deep as in any of the single stars.

Another way to gauge the rotation in these systems is to look for shifts of shell lines formed in the lower chromosphere during the atmospheric eclipses. Many investigators have done this. Griffin et al. (1990) argued they had found a displacement  $8.5 \text{ km s}^{-1}$ , corresponding to synchronous rotation, in a single precise observation of  $\zeta$  Aur. Earlier measurements of such displacements in  $\zeta$  Aur were not so clear-cut. McKellar & Petrie (1952) found that metallic lines in a 1950 eclipse gave displacements of only  $2.5 \text{ km s}^{-1}$ , possibly from rotation much slower than synchronous. Wilson & Abt (1954) found positive shifts in ingress and negative shifts in egress, as expected for rotation, but these shifts came with significant scatter and a temporal dependence different from that of rotation. McKellar & Butkov (1956) found that lines of ionized metals in the near-ultraviolet, which would be unblended with photospheric lines, followed the velocity of the K star to a few  $\text{km s}^{-1}$  in 1955–1956. Bauer (1994) reviewed the evidence for 31 Cyg, finding that velocities of the metallic shell lines (e.g., Fe I) were always about the same as the orbital velocity, and that in most cases the differences were not consistent with rotation. Because it has a grazing total eclipse, 32 Cyg might not be expected to show rotational displacements of metallic shell lines, and it does not (Wright & Hesse 1969).

These stars are clearly not rotating pseudosynchronously, or even synchronously *unless the single stars are rotating much faster than generally thought*, and we see no convincing evidence they are rotating significantly faster than single stars. In conducting this analysis, we have implicitly assumed that single K supergiants are not rotating, inasmuch as we have no theoretically calculated comparison spectra for such stars, nor would we trust them if we did. Analyses of bright giants (class II stars) by Gray & Toner (1986a) find rotation  $v \sin i \lesssim 3 \text{ km s}^{-1}$ , although Gray & Toner (1987) find higher rotation ( $v \sin i \lesssim 7 \text{ km s}^{-1}$ ) for the class Ib supergiants. Eventually it ought to be possible to directly test the idea that K supergiants are rotating at rates like those

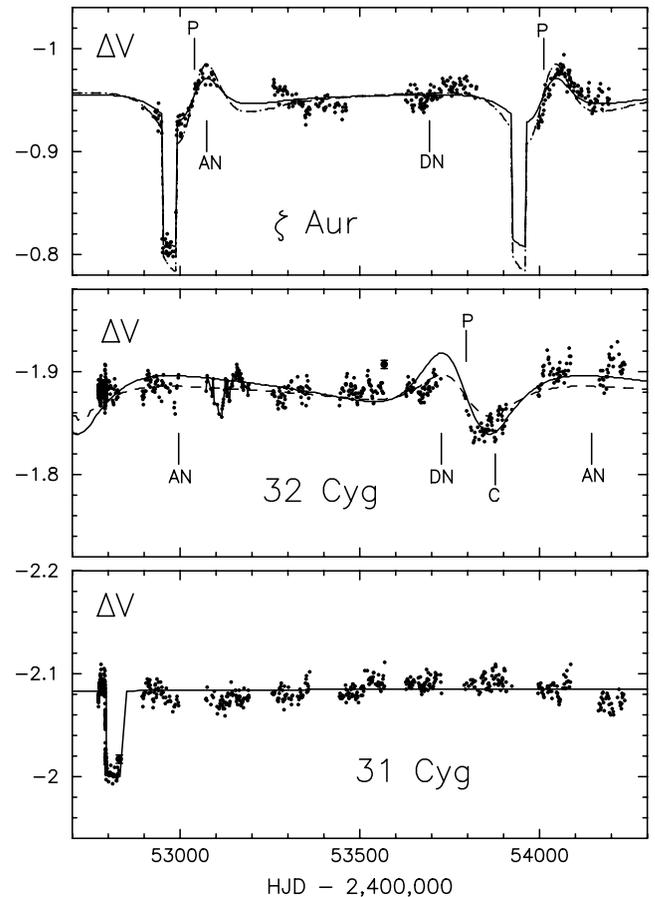


FIG. 3.—Light variation of the three binaries. These data for the  $V$  band show two types of intrinsic variation, the normally intermittent variation from pulsation and ellipsoidal variation, to at least some extent. The solid vertical lines mark times of periastron passage (“P”) and nodal passages (“AN” for ascending node and “DN” for descending node). The vertical line for 32 Cyg marked “C” shows the time of conjunction (primary eclipse). Thirty-one Cyg had a time of periastron passage at 52,373, outside the range plotted. The plotted curves represent the ellipsoidal variation calculated with the Wilson-Devinney code for these three systems. The ellipsoidal variation for  $\zeta$  Aur is quite prominent, peaking roughly a tenth phase after primary eclipse, as expected from the star’s orbital orientation. For 32 Cyg it contributes the prominent broad dip just before conjunction (primary eclipse). The rather sharp eclipse is lost in the noise of these data but is obvious at  $B$ . Pulsational variation is much harder to see in these data. There may be an example of it in the data around 53,125, which we have connected with line segments. The period would be 105 days.

inferred by Gray & Toner by using more precisely measured velocities of chromospheric shell lines in these binaries.

### 3.3. The Nonradial Pulsation from Ellipsoidal Light Variation

Guinan & McCook (1979) analyzed the light outside eclipse for 32 Cyg and concluded they had found the ellipsoidal variation of that star. Fredrick (1960; his Fig. 4) had previously detected variation looking like ellipsoidal variation on the even longer period of VV Cep.

Figure 3 shows the measured brightnesses of the three stars over the past 4.0 yr in the  $V$  band; data for  $B$  are similar. We seem to have detected the ellipsoidal variation of both  $\zeta$  Aur and 32 Cyg, along with its strong periastron effect around primary eclipse. The phase dependence for 32 Cyg is not as clear-cut as in  $\zeta$  Aur or in Guinan & McCook’s (1979) photometry, probably because of intrinsic variation. However, these two sets of photometry for 32 Cyg actually agree fairly well, especially close to periastron. Our data for 32 Cyg for the last two seasons illustrated have

unexpected shifts that mask the periastron effect to some extent. The light curve for 31 Cyg is incomplete, although it does show a sinusoidal wave ( $P = 1200$  days) over our period of observation that is not consistent with ellipsoidal variation. If this wave actually results from variation of their common comparison star, correcting for it would make the brightness of 32 Cyg more consistent with the calculated ellipsoidal variation.

The theoretical curves plotted in Figure 3 represent the ellipsoidal variation for these binaries calculated with the Wilson-Devinney code roughly for the elements given in Tables 1 and 4. They assume the stars are hardly rotating (0.1 synchronous) and incorporate the usual assumptions about the surface brightness, namely linear limb darkening ( $x = 0.80$ ) and Lucy's (1967) convective gravity darkening. However, it is not clear that a star with a driven nonradial pulsation could legitimately have its surface brightness distribution specified by gravity darkening. Gravity darkening is an equilibrium, diffusive theory that supposes time-scales much longer than the typical pulsation, driven or intrinsic. Furthermore, we know that pulsating stars can have a complicated, double-valued dependence of temperature on radius. If we assume to first order that *radially* pulsating stars are both apparently brighter and hotter when they are smaller, we might parameterize their variation of surface brightness,  $F$ , with gravity,  $\nabla\Omega \sim r^{-2}$ , as gravity darkening ( $F_{\text{bol}} \propto \nabla\Omega^g \propto r^{-2g}$ ). Then the gravity exponent,  $g$ , would be  $\sim 1.0$  for such a star to stay the same brightness ( $V$  magnitude) as it pulsates. Adiabatic pulsation would give a much higher effective value of  $g$ . Intrinsically driven pulsations of stars are far from adiabatic, and this leads to phase lags between the radial compression and the star's brightness, which depend on details of the star's structure and driving mechanism (e.g., Szabó et al. 2007). Nonradial pulsators are theoretically more complicated because horizontal motions relieve horizontal pressure variations, at least for nonradial pulsation much slower than the natural frequency of the star. Buta & Smith (1979) calculated the light variations of nonradial pulsators, finding that these horizontal adjustments greatly reduce the temperature variations predicted by Dziembowski's (1971) theory for adiabatic pulsation. In fact, they found that geometrical effects alone can explain most of the light variations of some actual nonradially pulsating stars. Furthermore, their calculations implied that the temperature variation might well be period dependent. What phase lags one might expect of nonradial pulsation is also open to conjecture (e.g., Townsend 2003). Theory obviously does not give us especially good guidance for predicting how temperature varies over the surface of a star with a driven pulsation.

We may get a better idea of how flux depends on gravity as a star pulsates by considering some actual pulsating stars. As a first stab at doing this, we have looked at analyses of  $\beta$  Cep stars, which are often suspected of having nonradial pulsations (e.g., Stamford & Watson 1977; Odell 1980). Calculations used to predict the complicated line profiles for assumed pulsation modes (e.g., Stamford & Watson 1976) often do not even include the effect of pulsation-induced effective temperature variations. However, we can get an estimate of that effect in an actual star by using Kubiak's (1972) analysis of BW Vul as an example. The values of effective temperature and effective gravity in his Table 6, admittedly somewhat double valued, give a slope of  $\Delta\log T_{\text{eff}}/\Delta\log g_{\text{eff}} \approx 0.18$ ,  $\text{org} \approx 0.7$ . This is larger than one might expect from extant theories of gravity darkening, but even contact components of Algol binaries, which seem to conform to the assumptions behind the theory, seem to require  $g \sim 0.5$ , somewhat larger than Lucy's 0.32 (Eaton 2008).

The data for  $\zeta$  Aur in Figure 3 and geometry derived by Bennett et al. (1996) are precise enough to give a crude test of the theory

incorporated into the Wilson-Devinney code. The solid curve is calculated for  $r_{\text{side}} = 0.163 (148 R_{\odot})^3$  with  $g = 0.32$ . The dashed curve represents the case  $r_{\text{side}} = 0.163$  with  $g = 1.0$ . The calculated phase dependence is excellent, meaning that there is no appreciable phase lag in the nonradial pulsation. The calculated amplitude is too small for  $g = 0.32$ , but somewhat large for  $g = 1.0$ . From this evidence, it appears the effective gravity darkening is  $g \approx 0.7-0.8$ . The theory thus seems to work to first order and restricts the radius to better than about 10%, as we judge for other calculations for  $R = 130$  and  $182 R_{\odot}$ .

The theory fails to predict the ellipsoidal variation of 32 Cyg, however, and this failure implies the radius must be much bigger than thought, as Guinan & McCook (1979) found in their own analysis, or the gravity darkening must be *much* larger than allowed by  $\zeta$  Aur. We have plotted the predicted variation for two cases in Figure 3,  $R = 217 R_{\odot}$  (*dashed line*) and  $R = 271 R_{\odot}$  (*solid line*), both for  $g = 0.7$ . The radius required to fit the amplitude with  $g = 0.7$  is  $r_{\text{side}} = 0.23 \pm 0.02 (260 \pm 20 R_{\odot})$ . The phase dependence of the periastron effect might be improved with a bigger eccentricity. However, the orbital elements are known very reliably from the radial velocities, and we therefore see no justification for using  $e$  as an aphysical fitting parameter in a light-curve solution.

Our rather crude analysis of the tidal distortion is about all anyone could expect to do without a much better theory of the surface brightness of these stars with driven pulsations.

### 3.4. Intrinsic Variation (Pulsational?)

Cool giants as a group seem to be pulsating, and the supergiant components of  $\zeta$  Aur binaries should be no exception. Our new photometry has very little evidence of pulsation beyond that driven by tides, but the radial velocities plotted in Figure 2 show unmistakable evidence of cyclic, if not periodic, variation in all three stars (see Table 5).

As for the photometry, it is difficult to detect long-term periods in any of these stars because of the strong, somewhat poorly modeled ellipsoidal light variation. There is obvious deviation from the calculated light curves in Figure 3. Thirty-two Cyg seems to have a coherent sinusoidal pulse of 105 day period around 53,125, which may have been a random pulsation. There seems to have been an anticorrelated change in the radial velocity, but the data sets did not overlap very well for that year, and they were rather noisy at the level of this effect. There also seems to be quasi-periodic variation of the brightness on shorter periods (e.g., near 53,500), but at the level of the noise in these data.

We may estimate the pulsational periods expected for these stars by using theoretical pulsational constants (e.g., Fox & Wood 1982) [ $Q = P(M/M_{\odot})^{1/2}(R/R_{\odot})^{-3/2} \propto P\sqrt{\rho}$ ] and mean densities derived from the information in Table 1. All three stars have roughly the same mean density, which is about one-tenth that of  $\alpha$  Tau. We would thus expect longer pulsational periods, by several times, than found in the normal giants. Values are  $Q/P = 1.34, 1.23, \text{ and } 1.35 \times 10^{-3}$ , respectively, for  $\zeta$  Aur, and 31 and 32 Cyg. The expected fundamental periods ( $Q \sim 0.08-0.18$ ) would be near 100 days, and the first overtone ( $Q \sim 0.03-0.04$ ), near 25 days. Alternatively, we can invert this process and calculate  $Q$ 's for observed periods to get the values listed in the column (5) of Table 5. These values are much larger than expected from any known pulsational mechanism, and are reminiscent of the unexplained long-period variations observed in many asymptotic giant branch stars (Wood et al. 2004; Hinkle et al. 2002).

<sup>3</sup> This  $r_{\text{side}}$  is the radius in the orbital plane perpendicular to the line between the centers of the two stars, measured in units of the orbital separation (semimajor axis).

Whether the periodic variations we see in the radial velocities represent pulsation or some other phenomenon is open to conjecture. Given the lack of *photometric* variation correlated with these variations of the radial velocities, we think it unlikely that the velocities result from pulsation of the stars. The aforementioned 105 day pulse of 32 Cyg has the right length for the radial fundamental, especially if we accept the reality of the apparent pulses at periods closer to the first overtone. This pulse has an amplitude  $\sim 0.5 \text{ km s}^{-1}$  peak-to-peak, giving an excursion of only  $1.9 R_{\odot}$ , or about 0.7%–1.1% of the radius of the star. Likewise, the apparent 125 day periodicity in the radial velocities of 31 Cyg, if a radial pulsation, implies that the radius changes by  $0.7 R_{\odot}$ , or 0.4%. Some of the longer periods detected in radial velocities, however, imply much bigger changes in radius, the 429 day periodicity of 32 Cyg ( $0.4 \text{ km s}^{-1}$  peak-to-peak), for example, corresponding to  $4.4 R_{\odot}$ . Furthermore, the singular excursion in the velocity of 31 Cyg near 53,700 corresponds to a change of  $>12\%$  of the radius, if pulsational.

If the changes in radius implied by cyclic radial velocities reflect radial pulsation, they must affect the timing of eclipses, and there is some evidence of this effect in the literature. When the radius of the star changes by 1%–2%, the duration of total eclipse must also change by a comparable amount, roughly 0.5 days in systems like  $\zeta$  Aur and 31 Cyg, so eclipse timings would give a way of testing for the existence of pulsations. The problem here is that eclipse timing is very difficult to measure in these systems, and the typical photometric analysis is compromised by being pieced together from data from many sources for the same eclipse. The best results seem to be for 31 Cyg, for which Stencel et al. (1984) determined times very precisely for the eclipses of 1962 and 1982. We can use their precise ephemeris to predict times in the 2004 eclipse observed with the APT and place a limit on how much the timing varies. Their  $t_{50}$  for ingress of 1982, extrapolated forward by two cycles, agrees with our observations to at least  $\pm 0.1$  days. This is a  $\pm 0.3\%$  change in the semiduration, or  $\sim 0.6 R_{\odot}$  in radius.

### 3.5. The Reflection Effect and Behavior of $H\alpha$

A reflection effect in these stars could take several forms: (1) heating of the atmosphere, detectable in changes of temperature-sensitive line ratios with phase; (2) increased ionization of the photosphere, detectable in weakening of very strong lines like  $\text{Na I D}$  and strengthening of lines of singly ionized species; and (3) higher ionization in the chromosphere, detectable as a change in  $H\alpha$ —probably an increase in its strength.

To look for effects of direct heating of the atmosphere, we have made composites of spectra of  $\zeta$  Aur and 32 Cyg at phases when the B star is behind the K star and when it is in front. If there are any differences in the photospheric absorption lines, they are very subtle. We looked for enhancements of such potentially sensitive lines as  $\text{Fe II } \lambda\lambda 6416.93$  and  $6432.68$ , finding a possible *very* slight enhancement with the hot star in front. The  $\text{Na I D}$  lines may have been a bit weaker in the irradiated spectrum of 32 Cyg, and  $H\alpha$  was definitely stronger in both stars, as seen in Figure 4.

Direct heating of the chromosphere by the B star could potentially change the strength of  $H\alpha$ . There are at least two ways  $H\alpha$  might vary in cool supergiants: (1) an overall change in the mass of the  $\sim$ hydrostatic chromosphere where the bulk of the line forms (e.g., Cram & Mullan 1979) and (2) an enhancement of the blue wing in the stellar wind (e.g., Mallik 1993), which is observed occasionally in supergiants (e.g., Smith & Dupree 1988; Eaton & Henry 1996).

Figure 4 shows the time/phase dependence of  $H\alpha$  in these three stars. We see variations much greater than the expected observational errors or the variations expected in normal (class III)

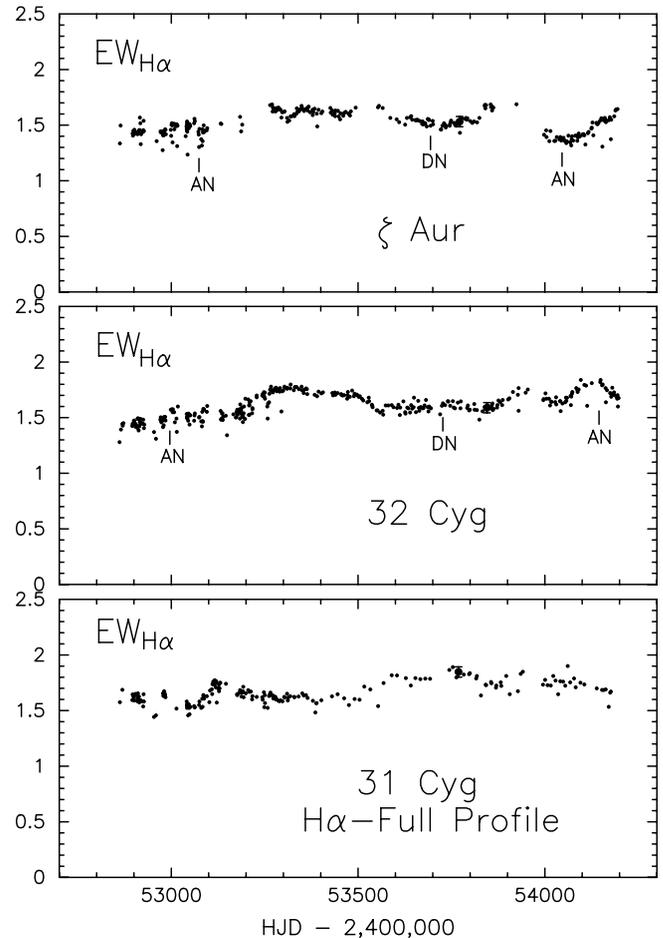


FIG. 4.— Variation of  $H\alpha$  in the three binaries. This is the equivalent width of the bulk of the profile,  $EW_1$  in  $\text{\AA}$ , which shows how the chromospheric absorption changes globally. Superior conjunction of the cool star, with its irradiated face in full view, occurs at 53,245 for  $\zeta$  Aur and 53,475 for 32 Cyg.

giants. The few single supergiants and bright giants measured by Eaton (1995) showed a much greater range of equivalent width, however, and the values for our three  $\zeta$  Aur components fall into that range. Furthermore, the three supergiants  $\epsilon$  Peg,  $\xi$  Cyg, and 63 Cyg have fluctuations of this amount during the same span of time. The phase dependence of  $H\alpha$  in the three binaries does not correlate neatly with their orbits. The strong enhancement seen in 32 Cyg over the range 53,200–53,550, for instance, occurred roughly when the cool star's irradiated face was pointing toward us, but its superior conjunction was near the end of this range, and the range does not coincide at all well with the time between the ascending and descending nodes, marked in the figure.

Our new data for these stars show very few of the eruptions/enhancements of winds seen in other supergiants. There are four instances of enhanced absorption in the blue wing of  $H\alpha$  apparent in Figure 5. An enhancement for  $\zeta$  Aur near 53,840 seems to be a real change in the stellar profile. The elevation for 31 Cyg near 53,800 is also probably stellar, as well. However, the high points for 32 Cyg marked “?” in the figure are probably telluric, and the high points for 31 Cyg near 54,000 could be also. On the other hand, the general enhancement in the absorption after 53,500 for 31 Cyg seems to reflect a genuine change in the profile.

## 4. DISCUSSION AND SUMMARY

The photometry and radial velocities of these three stars raise more questions about pulsation than they answer. In only one

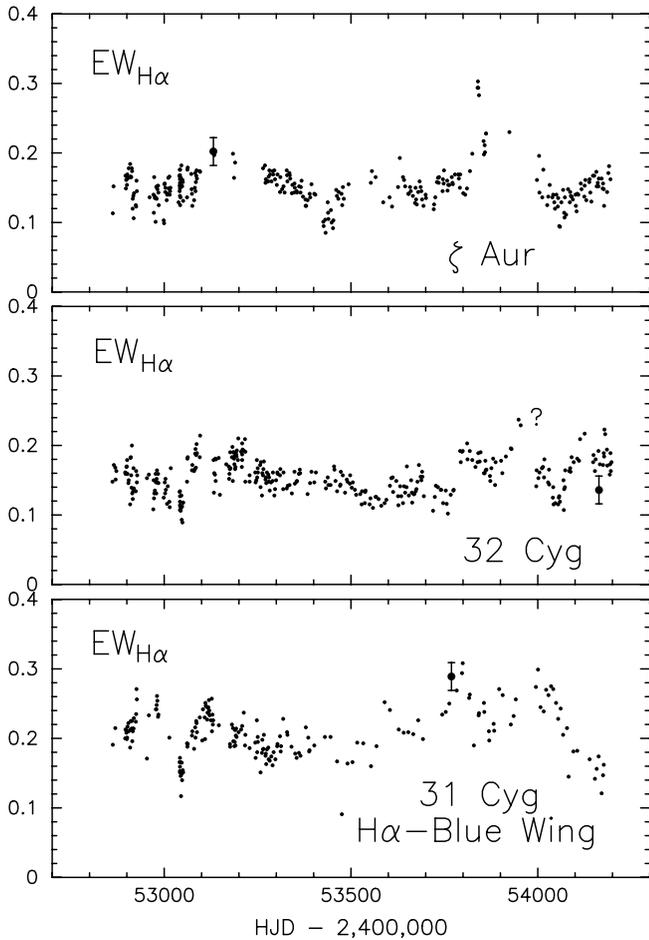


FIG. 5.— Variation of the blue wing of  $H\alpha$  in the three binaries. This is  $EW_{H\alpha}$  (in  $\text{\AA}$ ), the band that measures enhanced wind absorption in the blue wing of the profile beyond the velocity range of normal chromospheric absorptions. Such enhancements are amply documented in a number of cool supergiants, but there are few of them detected in these data.

very restricted case do we see something like a pulsation in both the brightness and radial velocity of the star. This pulse in 32 Cyg may have been a radial pulsation of the K star, and the same star shows flickering on shorter timescales, at the level of the photometric errors, that may be pulsation in overtones. The roughly coherent variation of 31 Cyg's radial velocity at 125 days may also reflect pulsation, but it is not accompanied by changes in the brightness. Radial pulsation at the 1%–2% level is consistent with eclipse timings in these stars.

The K supergiants in these systems fall in a part of the H-R diagram with rather low pulsation (e.g., Maeder 1980; Henry et al. 2000). Stars with lower masses are generally stable in their lower radial modes but become increasingly susceptible to pulsation in high overtones (Xiong & Deng 2007). Such high overtones are a possible source of the apparently random flickering of the rather stable K giants and supergiants.

One way to get a better idea of the level of any changes in radius from pulsation is to look critically at the timing and du-

ration of eclipses, as we have illustrated with three eclipses of 31 Cyg. We do not think the existing data are good enough to do this in any meaningful way. However, this approach should be possible with photometry from robotic telescopes, but it would take a communal effort over many years.

Other changes in both brightness and velocity are completely inconsistent with the known stability of light variation of these systems and with the expected pulsations of their K components. Especially perplexing is the 200 day drop in velocity of 31 Cyg, corresponding to a 12% change in the star's radius. This kind of change would be accompanied by changes of several days in the eclipse timing. It is much more likely to be a nonpulsational change in the circulation of the star's atmosphere, like the famous star patch in  $\zeta$  Boo A (Toner & Gray 1988). The range of photospheric velocity caused by granulation or other flows in cooler supergiants (Gray & Toner 1985, 1986a, 1986b, 1987) seems big enough ( $\sim 6\text{--}10\text{ km s}^{-1}$ ) to admit fluctuations at the level we are observing. However, once again we are thus reminded that “there are more things in heaven and earth than are dreamt of in [our] philosophy.”

All three of these stars had photometric variations on time-scales longer than expected for pulsation. Thirty-one Cyg showed a 1250 day sinusoidal variation in our photometry, covering  $\sim$ the length of our observations. There is no mechanism for producing this effect, and it may simply reflect variation of the comparison star. Both  $\zeta$  Aur and 32 Cyg also had variations in brightness beyond their ellipsoidal variation.

Cool components in these three classical systems seem to be rotating no faster than similar single supergiants. In contrast, Griffin et al. (1993) and Eaton & Shaw (2007) found evidence the chromosphere of 22 Vul is rotating *faster* than synchronously. This is a close binary in a circular orbit, which may have been a much closer, interacting system in a previous visit to the giant branch. Likewise, the supergiant component of the relatively close but eccentric binary HR 6902 (G9 II + B8 V) seems to be rotating even faster than pseudosynchronously (Griffin & Griffin 1986). These rotational velocities would seem to be an important clue to the evolutionary history of supergiant binaries once somebody becomes clever enough to interpret them.

We have detected the ellipsoidal variation and its periastron effect in two of the stars and used it to discuss how the driven non-radial pulsations in such a star should change the surface brightness. In this context, we question the use of the concept of gravity darkening in such stars and propose a methodology for determining an effective gravity darkening for such pulsations. We find the light variations of  $\zeta$  Aur require larger gravity darkening than predicted by Lucy's (1967) diffusive theory. Along these same lines, we may have detected a chromospheric reflection effect in the  $H\alpha$  strength.

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